

Slope & “Does the Line Pass Through?”

THE BIG IDEA: a straight line has the **same slope everywhere along it**. Pick any two points on the line and you get the same number. That single fact is what answers every “does the line also pass through this point?” question.

Slope: Rise over Run

how steep, and which way

Counting it straight off a graph

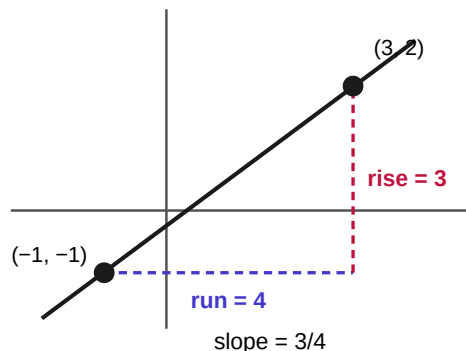
First point: (x_1, y_1)

Second point: (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

the RISE
how far UP

the RUN
how far ACROSS



WATCH OUT: start at the *same* point for both the top and the bottom of the fraction. Going from A to B and from B to A both work and give the same answer, but mixing them does not. From $(-3, -2)$ to $(2, 1)$: $(1 - (-2)) / (2 - (-3)) = 3/5$ is right, and $(1 - (-2)) / (-3 - 2) = -3/5$ is the classic wrong answer.

The Four Kinds of Slope

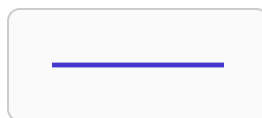
read the sign before you trust the number



POSITIVE
uphill to the right



NEGATIVE
downhill to the right



ZERO
flat, rise = 0, $m = 0$



UNDEFINED
run = 0, cannot divide

WATCH OUT: zero slope and undefined slope are opposites, not synonyms. A **flat** line has $m = 0$, because the rise is 0 and $0 \div 5 = 0$. A **vertical** line has **no slope at all**, because the run is 0 and you cannot divide by zero.

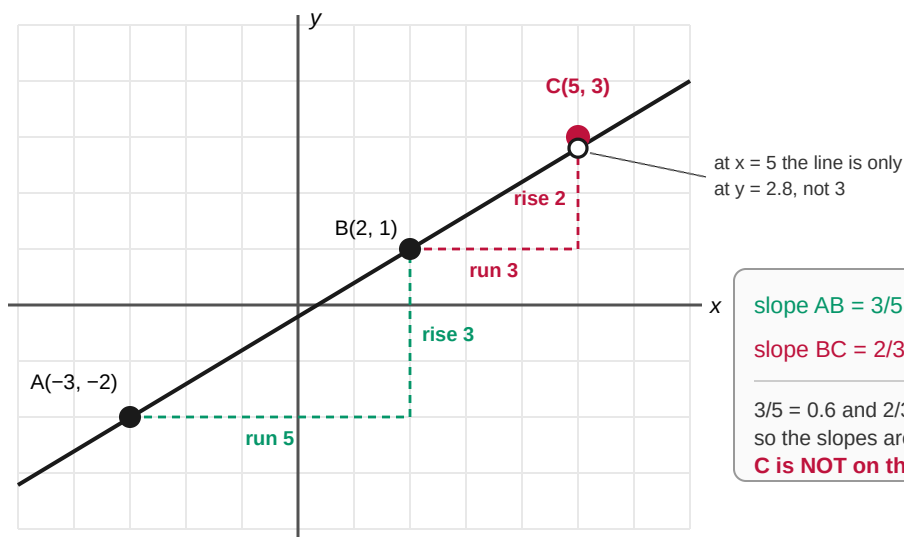
The Three-Step Test

this is problem 1-60

STEP	WHAT YOU DO	WHY IT WORKS
1	Find the slope from the first two points, the ones you know are on the line.	This is the line's true slope.
2	Find the slope from one of those points to the new point.	If the new point really is on the line, this must come out the same.
3	Compare. Same slope means yes, it is on the line. Different slope means no.	A line cannot change steepness partway along. Two different slopes means you have bent it, so the points cannot all lie on one straight line.

1-60 Worked

A(-3, -2), B(2, 1), does it pass through C(5, 3)?

slope AB = $3/5$ slope BC = $2/3$
 $3/5 = 0.6$ and $2/3 = 0.67$,
so the slopes are different.
C is NOT on the line.**Method 1: compare two slopes**A to B: $(1 - (-2)) / (2 - (-3)) = 3/5$ B to C: $(3 - 1) / (5 - 2) = 2/3$ Different, so **no**.**Method 2: build the rule and test it**Line through A and B: $y = (3/5)x - 1/5$ Put in $x = 5$: $y = 3 - 1/5 = 2.8$ C needs $y = 3$, so **no**.

HOW TO JUSTIFY IT: the problem says "justify your conclusion," so a bare no earns nothing. Write both slopes and one sentence: "The slope from A to B is $3/5$, but from B to C it is $2/3$. A straight line has the same slope everywhere, so C is not on line AB."

Finding Slope from Two Points

six shapes of answer

1 · POSITIVE WHOLE NUMBER

(1, 2) and (4, 8)

$$(8 - 2) / (4 - 1)$$

$$= 6 / 3 = 2$$

Uphill. Up 2 for every 1 across.

2 · NEGATIVE WHOLE NUMBER

(-1, 5) and (3, -3)

$$(-3 - 5) / (3 - (-1))$$

$$= -8 / 4 = -2$$

Downhill. The y value dropped, so the slope is negative.

3 · POSITIVE FRACTION

(-2, -5) and (4, -1)

$$(-1 - (-5)) / (4 - (-2))$$

$$= 4 / 6 = 2/3$$

Both y values are negative, but the slope is still positive: it went up.

4 · NEGATIVE FRACTION

(-4, 3) and (2, -6)

$$(-6 - 3) / (2 - (-4))$$

$$= -9 / 6 = -3/2$$

Reduce the fraction. Leaving it as $-9/6$ is not finished.

5 · ZERO SLOPE

(0, 4) and (5, 4)

$$(4 - 4) / (5 - 0)$$

$$= 0 / 5 = 0$$

Same height both times, so the line is flat.

6 · UNDEFINED SLOPE

(3, 1) and (3, 7)

$$(7 - 1) / (3 - 3)$$

$$= 6 / 0 = \text{undefined}$$

Same x both times, so the line is vertical. No slope exists.

Do All Three Points Lie on One Line?

same method as 1-60

EXAMPLE A · ANSWER IS YES

(1, 2), (3, 6), (5, 10)

First to second: $(6 - 2)/(3 - 1) = 4/2 = 2$

Second to third: $(10 - 6)/(5 - 3) = 4/2 = 2$

Same slope, so yes.

All three sit on one straight line of slope 2.

EXAMPLE B · YES, WITH NEGATIVES

(-2, 7), (1, 1), (4, -5)

First to second: $(1 - 7)/(1 - (-2)) = -6/3 = -2$

Second to third: $(-5 - 1)/(4 - 1) = -6/3 = -2$

Same slope, so yes.

Negative slopes work exactly the same way. Just keep the signs straight.

EXAMPLE C · ANSWER IS NO

(0, 0), (2, 3), (4, 7)

First to second: $(3 - 0)/(2 - 0) = 3/2$

Second to third: $(7 - 3)/(4 - 2) = 4/2 = 2$

Different, so no.

Close, but $3/2$ is 1.5 and 2 is 2. Close is not the same.

EXAMPLE D · PROBLEM 1-60

(-3, -2), (2, 1), (5, 3)

A to B: $(1 - (-2))/(2 - (-3)) = 3/5$

B to C: $(3 - 1)/(5 - 2) = 2/3$

Different, so no.

0.6 against 0.67. On a graph it looks like it might fit, which is exactly why you check with numbers.

WATCH OUT: compare the fractions properly before deciding. $3/5$ and $2/3$ look similar, so turn them into decimals or use a common denominator: $3/5 = 9/15$ and $2/3 = 10/15$. Now it is obvious they are different.

Five Ways This Goes Wrong

check these before you hand it in

THE MISTAKE	WHAT IT LOOKS LIKE	THE FIX
Flipping the fraction	run over rise, so $5/3$ instead of $3/5$	The rise (the y values) always goes on top. Say "rise over run" out loud.
Mixing the order	$(y_2 - y_1) / (x_1 - x_2)$	Whichever point you call first, use it first on <i>both</i> top and bottom. Mixing flips the sign.
Losing a negative	writing $1 - (-2) = -1$	Subtracting a negative adds: $1 - (-2) = 3$. Put the parentheses in every time.
Not reducing	leaving the answer as $-9/6$	Reduce to $-3/2$. An unreduced slope is an unfinished answer.
Calling it close enough	treating $3/5$ and $2/3$ as the same	Compare exactly. $3/5 = 9/15$ and $2/3 = 10/15$, so they are different and the answer is no.