

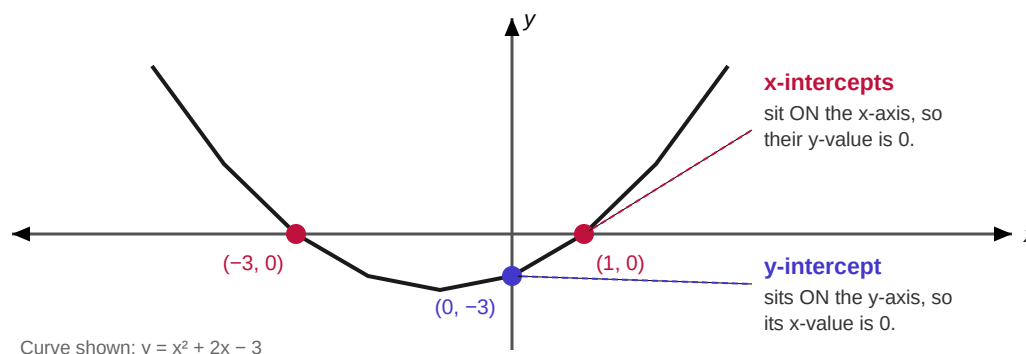
Intercepts, Symmetry & Vertex

THE ONE RULE THAT SETTLES EVERY INTERCEPT: an intercept is named for the axis it *crosses*, and on that axis the other variable is **zero**. x-intercept → set $y = 0$. y-intercept → set $x = 0$.

The Two Intercepts

where the graph meets an axis

YOU'RE ASKED FOR	SET TO 0	THEN YOU SOLVE FOR	WRITE THE ANSWER AS	HOW MANY CAN THERE BE?
x-intercept a.k.a. root, zero	$y = 0$	x. Needs real solving: factor, square root, or split the absolute value	$(x, 0)$ zero goes second	0, 1, or 2 (sometimes more). A parabola or V can miss the axis entirely.
y-intercept	$x = 0$	y. Just substitute and simplify, no solving needed	$(0, y)$ zero goes first	Exactly 1 , always. A function has one output at $x = 0$.



WATCH OUT: the most common mistake is setting the *wrong* variable to zero, because the names feel backwards. Say it out loud: **“x-intercept means I’m crossing the x-axis, and everything on the x-axis has $y = 0$.”**

Two Ways to Find an Intercept

read it off, or solve for it

A. From a graph: read it off

- Find where the curve **touches or crosses** an axis, and read the coordinates there.
- Write an **ordered pair**, not a bare number.
- Passing *near* a grid point is not an intercept; it must land on the axis.

B. From an equation: solve for it

- y-intercept:** replace every x with 0 and simplify. Fast.
- x-intercept:** replace y with 0, then solve what’s left.
- Finding y is substitution; finding x is real work. That asymmetry is normal.

Line of Symmetry & Vertex

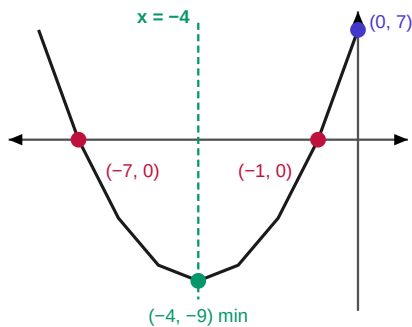
the fold line, and the turning point

FEATURE	HOW TO FIND IT	HOW TO WRITE IT
Line of symmetry	The vertical fold line. Average the two x-intercepts: $x = (x_1 + x_2) \div 2$. It always runs through the vertex.	As an equation , not a point: $x = -4$ (a whole vertical line)
Vertex	The turning point. Take the x from the line of symmetry and plug it back into the equation to get y .	As a point : $(-4, -9)$
Maximum or minimum	Which way does the graph open? Up → the vertex is the lowest point. Down → it is the highest point.	Name it: “minimum at $(-4, -9)$ ”

WATCH OUT: a line of symmetry is a *line*, so the answer is $x = -4$. Writing just “-4” or “ $(-4, -9)$ ” misses what was asked.

Worked Example: a parabola

$$y = x^2 + 8x + 7$$



Step by step

- y-intercept.** Set $x = 0$: $y = 0 + 0 + 7 = 7 \rightarrow (0, 7)$. Shortcut: it's always the constant term.
- x-intercepts.** Set $y = 0$ and factor: $(x + 7)(x + 1) = 0$, so $x = -7$ or $x = -1$.
- Line of symmetry.** Average them: $(-7 + -1) \div 2 = -4 \rightarrow x = -4$.
- Vertex.** Put -4 back in: $16 - 32 + 7 = -9 \rightarrow (-4, -9)$. Opens up, so it's a **minimum**.

A. X-INTERCEPTS
 $(-7, 0), (-1, 0)$

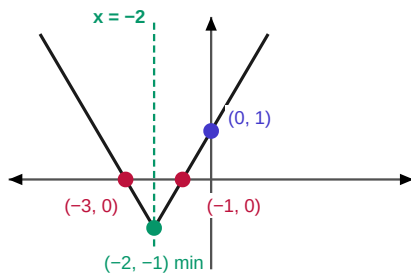
B. LINE OF SYMMETRY
 $x = -4$

C. Y-INTERCEPT
 $(0, 7)$

D. SPECIAL POINT
min at $(-4, -9)$

Worked Example: an absolute value V

$$y = |x + 2| - 1$$



Step by step

- y-intercept.** Set $x = 0$: $y = |0 + 2| - 1 = 1 \rightarrow (0, 1)$.
- x-intercepts.** Set $y = 0$, get the bars alone: $|x + 2| = 1$. Absolute value splits into **two** cases: $x + 2 = 1$ or $x + 2 = -1$, so $x = -1$ or $x = -3$.
- Line of symmetry.** The fold runs through the vertex: $(-3 + -1) \div 2 = -2 \rightarrow x = -2$.
- Vertex.** $y = |-2 + 2| - 1 = -1 \rightarrow (-2, -1)$, a **minimum**. No endpoints, both arms rise forever.

A. X-INTERCEPTS
 $(-3, 0), (-1, 0)$

B. LINE OF SYMMETRY
 $x = -2$

C. Y-INTERCEPT
 $(0, 1)$

D. SPECIAL POINT
min at $(-2, -1)$

WATCH OUT: the $+2$ inside the bars shifts the graph **left** 2, not right. Whatever is grouped with x moves the graph the *opposite* way.

“Find x” vs “Find y”

which one were you handed?

Before solving, ask: **which variable did the problem give me?** You were handed one and asked for the other, and that decides whether you *substitute* or *solve*.

THE PROBLEM GIVES YOU	WHAT TO DO	EXAMPLE	ANSWER
An input: an x , or $f(\dots)$ with a number inside	Substitute. Always exactly one answer.	$f(x) = x - 3 $, find $f(-4)$	$ -4 - 3 = -7 = 7$
An output: a y , or $f(x) = a$ number	Solve. Could be one, two, or no answers.	$f(x) = 3 - \sqrt{x}$, find x when $f(x) = 2$	$\sqrt{x} = 1$, so $x = 1$

When there are two answers

$f(x) = |x|$ with $f(x) = 3$ gives $x = 3$ **and** $x = -3$. Vs and parabolas hit the same height twice, once on each side of the line of symmetry.

When there is no answer

$f(x) = |x|$ with $f(x) = -5$ has **no solution**, because absolute value is a distance and can never be negative. Say *why*; don't just write “none.”

WATCH OUT: $f(-4)$ does not mean “ f times -4 .” It means “the output when the input is -4 ”, so every x in the rule gets replaced by (-4) , parentheses included.